Probabilistic versus Incremental Presynaptic Learning in Biologically Plausible Synapses

Francisco Javier Ropero Peláez[1] and Diego Andina[2]

[1] Center for Mathematics, Computation and Cognition, Federal University of ABC
francisco.pelaez@ufabc.edu.br
[2] Group for automation in signals and communication. UPM, Spain
d.andina@upm.es

Abstract. In this paper, the presynaptic rule, a classical rule for hebbian learning, is revisited. It is shown that the presynaptic rule exhibits relevant synaptic properties like synaptic directionality, and LTP metaplasticity (long-term potentiation threshold metaplasticity). With slight modifications, the presynaptic model also exhibits metaplasticity of the long-term depression threshold, being also consistent with Artola, Brocher and Singer's (ABS) influential model. Two asymptotically equivalent versions of the presynaptic rule were adopted for this analysis: the first one uses an incremental equation while the second, conditional probabilities. Despite their simplicity, both types of presynaptic rules exhibit sophisticated biological properties, specially the probabilistic version.

Keywords: Metaplasticity, ABS rule, NMDA channel, BCM rule.

1 Introduction

Synapses are neural structures that modulate presynaptic activity, converting this activity into a higher or lower postsynaptic activation (voltage). The magnitude that relates postsynaptic voltage with presynaptic activity is called synaptic efficiency or synaptic weight.

To model brain learning, it is necessary to start with a plausible hypothesis of synaptic weight modification. According to the plasticity curve in Figure 1.a, synaptic modification depends on the value "a" of postsynaptic activation [1]. If the value (voltage) of this postsynaptic depolarization is above the Long-Term Potentiation (LTP) threshold (white dot), a positive variation of synaptic weight, that is, synaptic potentiation, occurs. If the value of postsynaptic activation is between the LTP threshold and the Long-Term Depression (LTD) threshold (black dot), a negative variation of synaptic weight, that is, synaptic depression, takes place. Below the LTD threshold, neither potentiation nor depression occurs.

Another important property related to synaptic plasticity is metaplasticity [2] which is a homosynaptic property. Metaplasticity is related to the change in the shape of the plasticity curve by either a change of the regime (average over

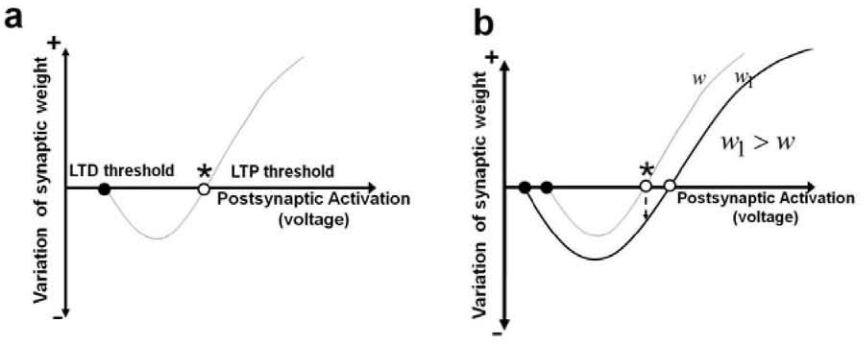

Fig. 1. Synaptic weight modification of a biological synapse according to its postsynaptic activation. (a) For values lower than the LTD threshold (black dot) no variation of synaptic weight takes place. For values between the LTP threshold (white dot) and the LTD threshold, the variation of the synaptic weights is negative, i.e. the synaptic weight decreases. For values higher than the LTP threshold, the synaptic weight increases. (b) Synaptic metaplasticity: Curves corresponding to higher values of ω have a higher LTP threshold and a lower LTD threshold.

time) of postsynaptic activations $\hat{a}$, or a concomitant change in synaptic weight ω (Fig. 1.b).

Let us consider the curve of figure 1.a in which the independent variable is the postsynaptic activation and where other parameters such as $\hat{a}$ or ω are kept constant. For example, let us evaluate the variation of weight for a specific value of postsynaptic activation by setting the postsynaptic activation (see asterisk in the curve of Fig. 1.a) to the value of the LTP threshold. In this case, no variation of weight is observed. However, if the regime of synaptic activations $\hat{a}$ is changed, so that ω reaches a higher value ω_1, metaplasticity is manifested by the modification of the shape of the curve as shown in figure 1.b. In this new situation, if the same postsynaptic voltage is applied as before (see asterisk in the curve), instead of obtaining a null variation of weight, the new curve yields a negative variation (a decrement) of weight (see arrow).

In the first articles that study metaplasticity [2], metaplasticity was related to the rightward shift of LTP-thresholds for higher $\hat{a}$'s or ω's. More recent studies [3] showed that the LTD thresholds diminish in the same circumstances (Figure 1.b). In summary, once synapses are positively primed (i.e. there is an increment in weight), the interval between thresholds broadens, thereby favouring subsequent synaptic depression.

Along the years, different mathematical models of synaptic computation have been proposed (see Figure 2.a). In the classical Hebb model [4], the curve relating the increment of synaptic weight to postsynaptic activation is a straight line without synaptic depression. In Sejnowski's covariance model [5][6], regions of potentiation and depression are separated by a LTP threshold (white dot).

Artola, Brocher and Singer's [8] extended model (ABS model, Figure 2.b) is not analytical, as those just discussed, but is based on empirical experimental

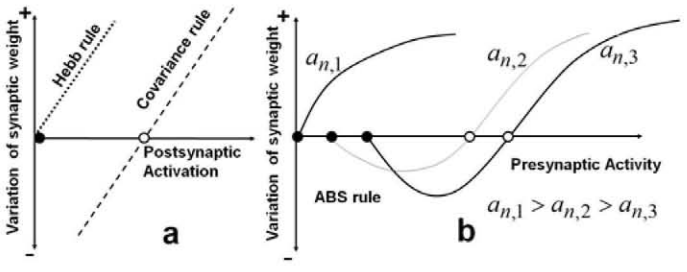

Fig. 2. Different models of synaptic plasticity and metaplasticity. (a) Hebb rule and Sejnowski's covariance rule. (b) The ABS qualitative model by Artola, Brocher and Singer. a_n, represents the postsynaptic activation due to neighbouring synapses to the one under consideration. For higher values of a_n, the curve is shifted to the left.

data. In the ABS model, LTP and LTD thresholds shift to lower values for higher levels of the activation of neighbouring synapses a_n. For example, if the presynaptic activity (action potential frequency) of a specific synapse is negligible, and activations from neighbouring synapses are high, as in curve $a_{n,1}$, synaptic potentiation occurs in that synapse.

In the following sections, the incremental and probabilistic versions of the presynaptic rule [9] will be analyzed in order to assess their biological plausibility in the light of the mentioned synaptic properties (i.e. metaplasticity of both LTP and LTD thresholds, and the ABS rule).

In the incremental version of the presynaptic rule, the variation of the synaptic weight ω is calculated as:

$$\triangle \omega = \xi I(O - \omega) \tag{1}$$

where I and O (in uppercase) are the presynaptic and postsynaptic action potential frequencies or, if normalized, the presynaptic and postsynaptic action potential probabilities respectively. ξ is a small positive constant. According to Grossberg[9] and Minai [10] this incremental version of the presynaptic rule is asymptotically equivalent to the following probabilistic expression:

$$\omega = P(\frac{o}{i}) \tag{2}$$

which is regarded to be "the probabilistic version of the presynaptic rule". $P(\frac{o}{i})$ is the conditional probability of a postsynaptic action potential, o, given a presynaptic action potential i (notice that binary action potentials are represented in lowercase while action potential frequencies or probabilities are in uppercase).

2 Methods

The two versions of the presynaptic rule correspond respectively to the following mathematical models:

In the first model a rate-code neuron is used, in which the output of the neuron is a number expressing its firing frequency or probability. In this model the incremental rule of synaptic plasticity is applied.

In the latter, a spiking neuron model is used, in which the output is binary, and triggers according to the calculated firing probability. In this case, the probabilistic version of the presynaptic rule is applied.

In the first rate-code model, the action potential probabilities of presynaptic and postsynaptic neurons are respectively written I and O (in upper case, as explained before). The synaptic weight ω relates I and the excitatory postsynaptic potential, e, at a specific synapse j.

$$e_j = \omega_j I_j \tag{3}$$

The neuron's activation, a, is obtained after summing the postsynaptic potentials of all synapses:

$$a = \sum_{j=1}^{n} e_j = \sum_{j=1}^{n} \omega_j I_j \tag{4}$$

Afterwards, the action potential probability is calculated as a non-linear (sigmoidal) function of a: $O = f(a)$.

The rate code model is simplified by working in the linear part of the sigmoidal curve.

In the second mathematical model, the so called spiking output model, synaptic weights are calculated taking into account the degree of correlation between presynaptic and postsynaptic action potentials considered as binary outputs. When an action potential is triggered it is represented by a binary one, whereas the absence of an action potential is represented by a binary zero. The output of the presynaptic neuron ($bit = 1$ or $bit = 0$) is denoted by i (lowercase) while the postsynaptic output ($bit = 1$ or $bit = 0$) is denoted by o. The probabilistic version of the presynaptic rule for obtaining synaptic weights is:

$$\omega = P(\frac{o}{i}) = \frac{n(o \cap i)}{n(i)} \tag{5}$$

where $n(\ )$ in the numerator counts the number of coincidences between presynaptic and postsynaptic binary outputs.

The calculus of e and a is the same as for the rate code output, being ω a conditional probability:

$$e_j = P(\frac{o}{i_j})I_j \tag{6}$$

Note that I_j (uppercase) is also a probability and not a binary output. In the spiking output model, o is generated according to O, which is obtained by applying to "a" a logistic function $f(a)$ of the type:

$$O = \frac{1}{1 + e^{\frac{-(a-s)}{T}}} \tag{7}$$

where parameter T is usually called temperature and regulates the inclination of the curve. The usual procedure for generating an "o=1" with probability O, is generating a random number between o and 1. If the random number is lower than O a binary "o=1" is produced. Parameter s is the shift of the sigmoid. In the following sections we arbitrarily set T to 0.066 and s to 0.5.

3 Results

At molecular levels, plasticity is related to the amount of Ca^{2+} entering postsynaptic NMDA channels (Artola [1]). When calcium is above a certain threshold, metabolic events lead to synaptic potentiation. Conversely, lower calcium levels yield synaptic depression. Without calcium, neither potentiation nor depression takes place. Notice that calcium entrance depends on the presence of presynaptic action potentials. Therefore, presynaptic action potentials are necessary for homosynaptic plasticity. The need of a presynaptic action potential for allowing homosynaptic plasticity will be called "synaptic directionality".

Several facts related to NMDA channel operation should be taken into account in order to perform a realistic model of plasticity in NMDA channels. We consider two cases: the case in which plasticity is restricted to a single synapse (homosynaptic plasticity) and the case in which there is some influence from other synapses over the one under study (heterosynaptic plasticity).

A0. Homosynaptic plasticity. We will take into account the three following facts:

A1. Synaptic directionality. A presynaptic action potential is necessary to produce any positive or negative change in synaptic plasticity.

A2. Metaplasticity of the LTP threshold. The greater the synaptic weight, the higher the LTP threshold.

A3. Magnesium leakage in NMDA channels: When neurotransmitters are bound in AMPA and NMDA channels, ions like Na^+ and Mg^{2+} enter through the channels, and produce the postsynaptic voltage. The greater the number of channels, the higher is this postsynaptic voltage. However, once NMDA channels are opened, there is not only an entrance of Ca^{2+} ions, but also an outward leakage of Mg^{2+} and K^+ ions, that diminishes the expected postsynaptic voltage.

B0. Heterosynaptic plasticity. Two or more nearby synapses are involved, and we consider the following additional facts that are not present in homosynaptic plasticity:

B1. The postsynaptic potential in the synapse under study is incremented by the postsynaptic potential of nearby synapses.

B2. Metaplasticity of the LTD threshold. The higher the weight, the lower the LTD threshold (Mockett et al [3].

B3. The ABS rule.

3.1 Biological Plausibility of Presynaptic Rule Incremental Version

Let us recall the incremental version of the pre-synaptic rule:

$$\triangle \omega = \xi I(O - \omega) \tag{8}$$

Notice that, consistently with the property of synaptic directionality (A.1), if "I" is zero, neither potentiation, nor depression takes place, so term "I" can be regarded as an "allowance term". According to *Methods*, "a" and O can be made proportional in the linear part of the logistic function so that, according to equation 4, I can be replaced by $\frac{O}{\omega}$ or $\frac{a}{\omega}$ in the homosynaptic case, so that:

$$\triangle \omega = \xi(\frac{a}{\omega})(a - \omega) = \xi[(\frac{a^2}{\omega}) - a] \tag{9}$$

which is represented in Fig. 3.a, where the potentiation and depression regions described by Artola et al [1] are present. The curves also exhibit metaplasticity of the LTP threshold.

In the case of heterosynaptic plasticity, the postsynaptic activation a is the sum of the contribution of the synapse being evaluated, ωI, and the potential from nearby synapses, a_n. Recalling property B.1.

$$a = \omega I + a_n \tag{10}$$

Isolating I and taking into account that this term is always positive or zero, the incremental presynaptic rule (Eq. 8) becomes:

$$\triangle \omega = \xi \frac{[a - a_n]_+}{\omega}(a - \omega) \tag{11}$$

where $[a - a_n]_+$ indicates the positive component of $a - a_n$, or zero if $a - a_n$ is negative. Setting a_n to a positive value, as for example 0.3, last expression gives rise to the more realistic curve of figure 3.b in which for low postsynaptic activations ($a < a_n$) neither potentiation nor depression takes place. The curve also exhibits a LTD threshold for $a = a_n$.

This same equation is consistent with the ABS rule in which the horizontal axis represents the presynaptic activity, I, instead of a. Substituting equation 10 in equation 9, equation 9 is expressed in terms of I as:

$$\triangle \omega = \xi I(\omega I + a_n - \omega) \tag{12}$$

which is the curve represented in Fig. 3.c showing a leftward shift of the LTP threshold for greater values of a_n. At the same time, the curve exhibits the uncommon properties described by Artola and Singer [8] in which synaptic potentiation occurs with negligible values of I in the case of very large values of a_n.

Finally, it is possible to take property A.3 into account in the case of heterosynaptic plasticity (see also property B.2).

According to this, the greater the number of AMPA and NMDA channels, that is, the greater the synaptic weight, the greater is also the leakage of K^+ and Mg^{2+} ions through these channels. Representing this leakage by a subtractive term proportional to the weight, the postsynaptic activation is calculated as:

$$a = I\omega - k\omega + a_n \tag{13}$$

In this case the presynaptic rule is expressed in terms of a as:

$$\triangle\,\omega = \xi[\frac{a - a_n}{\omega} + k]_+(a - \omega) \tag{14}$$

yielding fig 3.d, which is consistent with the property of metaplasticity of the LTD threshold.

3.2 Biological Plausibility of the Probabilistic Version of Presynaptic Rule

In previous sections, we simplified the mathematical description of a neuron by making the assumption of linearity in neuron activation function. In this section,

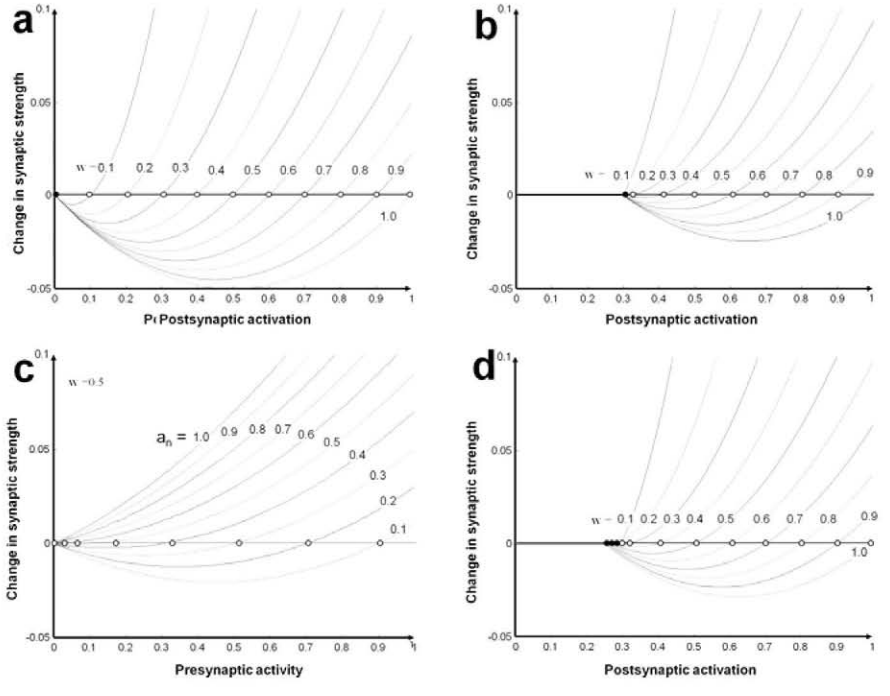

Fig. 3. Metaplasticity and the ABS rule according to the incremental version of presynaptic equation. (a) LTP threshold metaplasticity according to Eq. 9: the different curves are obtained for different values of synaptic weight ω. (b) LTP threshold metaplasticity considering heterosynaptic influence according to Eq. 11. (c) ABS theory according to presynaptic rule incremental version in equation 12. Presynaptic activity is shown in the horizontal axis and the curves are obtained for different values of a_n. The weight was $\omega = 0.5$ (d) Metaplasticity of the LTD threshold according to equation 14.

the neuron output, o, is binary and the probability of a unitary output is given by a non-linear logistic activation function, so that:

$$o = f^*(a) \tag{15}$$

f^* being a function that yields a binary 0 or 1 according to the probability given by O, which is the output given by the sigmoid function f where $O = f(a)$. Here, the variation of synaptic weight $\omega - \omega_0$ is calculated for each activation a and for each initial weight ω_0 by using the probabilistic version of the presynaptic rule: $\omega = P(\frac{o}{i})$. Binary output o is calculated according to Eq. 15 whereas binary input i is calculated from probability I, where in the homosynaptic case, $I = a/w$.

When including a_n (the postsynaptic activation due to other synapses), I is obtained from equation 10 as:

$$I = \frac{[a - a_n]_+}{\omega} \tag{16}$$

Fig. 4.a is the representation of this case. The influence of a_n, that was arbitrarily set to $a_n = 0.3$ in the example, is observed in the rightward shift of 0.3 units of the curves in Figure 4.a.

When property A.3 is taken into account, the presynaptic probability, I, is calculated as:

$$I = [\frac{a - a_n}{\omega} + k]_+ \tag{17}$$

Fig. 4.b corresponds to this case. There is a leftward shift of the LTD threshold with greater values of w (i.e. LTD metaplasticity). Without a_n, LTD metaplasticity is not present. This property is only manifested in the case of heterosynaptic plasticity (property B.2).

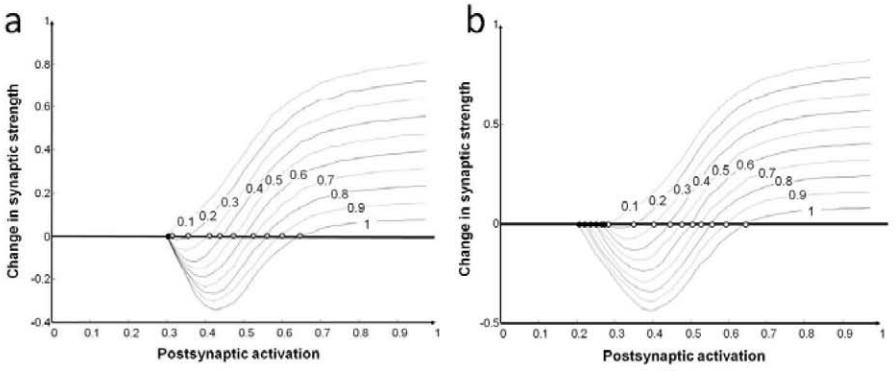

Fig. 4. Plasticity curves calculated according to the probabilistic presynaptic equation using a non-linear activation function (sigmoid). (a) In this case, the curves exhibit LTP threshold metaplasticity considering heterosynaptic influence according to Eq. 16. (b) Metaplasticity of the LTD threshold according to equation 17 and considering property A.3.

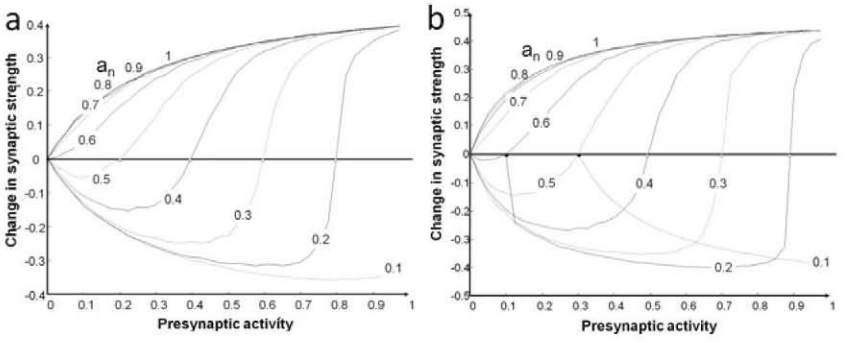

Fig. 5. Synaptic weight variation in terms of presynaptic activity (with the activation of neighbouring synapses an as a parameter). Presynaptic probabilistic equation with the ABS rule (a) Using a probabilistic presynaptic equation with a non-linear activation function (sigmoid).(b) Taken property A.3 into account.

To test whether the conditional probability model of synaptic plasticity is consistent with the ABS rule, the presynaptic probability I is represented in the horizontal axis. To obtain $P(\frac{o}{i})$, o is given by $o = f^*(a)$, a being a function of both I and a_n, according to equation 10. Fig. 5.a seems to encompass more consistently the ABS rule (Fig. 2.b) than our previous attempt in Fig. 3.c. If property A.3 is taken into account (so that a is obtained from Eq. 13 Fig. 5.a become Fig. 5.b, which is even more consistent with the ABS rule (compare with Fig. 2.b).

4 Conclusion

In this work, we have analyzed the presynaptic rule of synaptic plasticity in the light of recent biological findings such as synaptic directionality, metaplasticity of potentiation and depression thresholds, and the influence of nearby synapses (the so called "ABS" rule) over the synapse under study.

The presynaptic rule is consistent with synaptic directionality and LTP metaplasticity, in the case of homosynaptic plasticity, and with LTD metaplasticity and the ABS rule, in the case of heterosynaptic plasticity.

All these properties were studied with: a) the incremental equation of the presynaptic rule, assuming linearity of the activation function around the average activation set-point, and with b) the probabilistic version of presynaptic rule in which a logistic activation function was utilized. While both analyses were consistent with biological findings, the latter seemed to more accurately encompass the qualitative features of experimental curves.

References

1. Artola, A., Brcher, S., Singer, W.: Different voltage-dependent thresholds for inducing long-term depression and long-term potentiation in slices of rat visual cortex. Nature 347, 69–72 (1990)
2. Abraham, W.C., Bear, M.F.: Metaplasticity: the plasticity of synaptic plasticity. Trends Neurosci. 19, 126–130 (1996)
3. Mockett, B., Coussens, C., Abraham, W.C.: NMDA receptor-mediated metaplasticity during the induction of long-term depression by low-frequency stimulation. European Journal of Neuroscience 15, 1819–1826 (2002)
4. Hebb, D.O.: The organization of behaviour: A neurophysiological theory. John Wiley and Sons, New York (1949)
5. Sejnowski, T.J.: Storing covariance with non linearly interacting neurons. J. Math. Biol. 4, 303–321 (1977)
6. Sejnowski, T.J., Chattarji, S., Stanton, P.K.: Homosynaptic long-term depression in hippocampus and neocortex. Seminars in the Neurosciences 2, 355–363 (1990)
7. Bienenstock, E.L., Cooper, L.N., Munro, P.W.: Theory for the development of neuron selectivity: orientation specificity and binocular interaction in visual cortex. J. Neurosci. 2(1), 32–48 (1982)
8. Artola, A., Singer, W.: Long-term depression of excitatory synaptic transmission and its relationship to long-term potentiation. Trends Neurosci. 16, 480–487 (1993)
9. Grossberg, S.: Adaptive pattern classification and universal recoding: I. Parallel development and coding of neural feature detectors. Biological Cybernetics 23, 121–134 (1976)
10. Minai, A.A.: Covariance learning of correlated patterns in competitive networks. Neural Computation 9, 667–681 (1997)
11. Abraham, W.C., Tate, W.P.: Metaplasticity: A new vista across the field of synaptic plasticity. Progress in Neurobiology 52, 303–323 (1997)
12. Turrigiano, G.G.: Homeostatic plasticity in neuronal networks: the more things change, the more they stay the same. Trends Neurosci. 22(5), 221–227 (1999)